Solution to Problem 1 problem set no. 1

1. Verify that $\Psi(t) = \sum_n C_n e^{-iE_n t/\hbar}\psi_n$ is a solution to the time-dependent

   Schrödinger equation, assuming that the Hamiltonian is time independent.

The time-dependent Sch. equation is

$$i\hbar\dot{\Psi}(t) = H\Psi(t)$$

$$\text{and thus } H\Psi(t) = \sum_n C_n e^{-iE_n t/\hbar} H\psi_n = \sum_n C_n e^{-iE_n t/\hbar} E_n\psi_n \qquad (1)$$

Note we have made use of the fact that $H\psi_n = E_n\psi_n$. (This is crucial to the proof.)

Directly doing the time differentiation

$$i\hbar\dot{\Psi}(t) = i\hbar\sum_n C_n\left(-i\frac{E_n}{\hbar}\right)e^{-iE_n t/\hbar}\psi_n = \sum_n C_n e^{-iE_n t/\hbar} E_n\psi_n. \qquad (2)$$

Remember that $-i^2 = 1$. Thus, (1) and (2) are equal and so we have verified that $\Psi(t) = \sum_n C_n e^{-iE_n t/\hbar}\psi_n$ is a solution to the td Schroedinger equation.