Basics of QM

Classical Recap

$$H = \frac{p^2}{2m} + V(q)\,;\ \dot{q} = \frac{\partial H}{\partial p};\quad \dot{p} = -\frac{\partial H}{\partial q} = -\frac{\partial V}{\partial q}$$

Quantum Mechanics

$$H_{op} = \frac{p_{op}{}^2}{2m} + V(q)\,;\ \ p_{op} = -i\hbar\frac{\partial}{\partial q}$$ **- one degree of freedom**

$$H_{op} = -\frac{\hbar^2}{2m}\quad {}^2 + V(q);\quad p_{op} = -i\hbar\quad = -i\hbar\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial y}\hat{z}\right)$$ **- 3dof**

The Schrödinger equation(s)

$$i\hbar\frac{\partial \Psi}{\partial t} = H_{op}\Psi$$ **- Most general, time dependent**

$$\Psi(q,t) = e^{-iH_{op}t/\hbar}\Psi(0) = (1 - H_{op}t + \ldots)\Psi(q,0)$$ **- Formal solution for time independent Hamiltonians**

For special class of wavefunctions, which are termed the eigenfunctions

$$H_{op}\psi_n = E_n\psi_n$$ **- Time independent**

This set of functions have the following properties:

$$\int dq\psi_n^*\psi_m = \delta_{n,m} = 1\ if\ \ m = n,\quad 0\ if\ \ m \neq n$$ **- orthonormal**

Dirac invented notation for this integral: $\langle \psi_n | \psi_m \rangle = \delta_{n,m}$

Then a special solution to the td Schrödinger equation is

$\Psi_n(t) = e^{-iE_n t/\hbar}\psi_n$.

The general solution is

$\Psi(t) = \sum_n C_n e^{-iE_n t/\hbar}\psi_n$**, where the expansion coefficients are given by**

$$C_n = \langle \psi_n | \Psi(0) \rangle$$

Matrix elements and Expectation values:

$$A_{m,n} = \langle \psi_m | A_{op}\psi_n \rangle = \int dq(\psi_m)^* A_{op}\psi_n$$

Hermitian operator A_{op}:

$$= \langle A_{op}\psi_m | \psi_n \rangle = \int dq (A_{op}^{+}\psi_m)^* \psi_n$$

$$= \int dq (A_{op}\psi_m)^* \psi_n = [\int dq (A_{op}\psi_m)(\psi_n)^*]^*$$

$$But \langle A_{op}\psi_m | \psi_n \rangle = \langle \psi_n | A_{op}\psi_m \rangle^*, so$$

$$A_{m,n} = (A_{n,m})^*$$